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MA110 - Engineering Mathematics-1
Problem Sheet - 10

Triple Integrals in Rectangular, Cylindrical and Spherical Coordinates

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1. Using triple integrals in Cartesian Coordinates, find the volume of the sphere
$$x^2 + y^2 + (z-4)^2 = 4.$$

2. Find the volume of the wedge cut from the cylinder $x^2 + y^2 = 1$ by the planes $z = -y$ and $z = 0$.

3. Find the volume of the region bounded in back by the plane $x = 0$, on the front and sides by the parabolic cylinder $x = 1 - y^2$, on the top by the paraboloid $z = x^2 + y^2$, and on the bottom by the xy-plane.

4. Find the volume of the solid enclosed by $z = x^2 + y^2$ and $z = 9$ using triple integrals in Cartesian coordinates.

5. Integrate $\sqrt{x^2 + y^2 + z^2}\ e^{-(x^2+y^2+z^2)}$ over the region bounded by the spheres$x^2 + y^2 + z^2 = a^2$, $x^2 + y^2 + z^2 = a^2$, $a > b > 0$. Write the integral in all three co-ordinate systems and evaluate the volume using the simplest one.

6. Find the volume of the region obtained by intersecting the ellipsoid $x^2 + 2y^2 + 2z^2 \leq 10$ and the cylinder $y^2 + z^2 \leq 1$.

7. Obtain the volume of the region enclosed by the cones $z = \sqrt{x^2 + y^2}$ and $z = 1 - 2\sqrt{x^2 + y^2}$.

8. Write the triple integral to find the volume of a cuboid of dimension a in both cylindrical and spherical coordinates.

9. Find the volume of the solid bounded above by the sphere $x^2 + y^2 + z^2 = 5$ and below by the paraboloid $x^2 + y^2 = 4z$. Write the integral in both cylindrical and spherical coordinates. (*Ans:* $\frac{2\pi(5^{3/2}-4)}{3}$)

10. Using triple integral in spherical coordinates in the order $d\rho d\phi d\theta$, find the volume of the cylinder $x^2 + y^2 = 1$ between the planes $z = 1$ and $z = 2$. (*Ans:* π)

11. Using triple integral in cylindrical coordinates in the order $dzdrd\theta$, find the volume of the sphere $x^2 + y^2 + z^2 = 3$ between the planes $z = \frac{\sqrt{3}}{2}$, $z = -\frac{\sqrt{3}}{2}$.

12. Find the volume of the solid enclosed by the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 2$. (*Ans:* $7\pi/3$)

13. Find the volume of the solid inside both the spheres $\rho = 2\sqrt{2}\cos\phi$ and $\rho = 2$. (*Ans:* $\frac{2(8-3\sqrt{2})\pi}{3}$)

14. Write the triple integral in cylindrical coordinates to evaluate the volume of the solid in first octant bounded above by the paraboloid $z = 4 - x^2 - y^2$ and aterally by the cylinder $x^2 + y^2 = 3x$ using following order of integrations and evaluate the: (a) $dzdrd\theta$, (b) $d\theta dzdr$, (c) $drdzd\theta$. (*Ans:* $\frac{5\pi}{4}$)

15. Write a triple integral representing the volume of the region between spheres of radius 1 and 2, both centred at the origin. Include limits of integration but do not evaluate. Use: (a) Spherical coordinates. (b) Cylindrical coordinates.

16. A homogeneous solid sphere of radius a is centred at the origin. For a section bounded by $\theta = -\alpha$ and $\theta = \alpha$, find the average distance from the z axis. (*Ans:* $\frac{3\pi}{16}$)

17. Consider the integral

$$\int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} dz\, dy\, dx$$

Rewrite the above integral as an equivalent iterated integral in the five other possible orders.

18. Find the volume of the wedge cut from the cylinder $x^2 + y^2 = 1$ by the planes $z = -y$ and $z = 0$.

19. Find the volume of the region bounded in back by the plane $x = 0$, on the front and sides by the parabolic cylinder $x = 1 - y^2$, on the top by the paraboloid $z = x^2 + y^2$, and on the bottom by the xy-plane.

20. Let D be the region bounded below by the plane $z = 0$, above by the sphere $x^2 + y^2 + z^2 = 4$ and on the sides by the cylinder $x^2 + y^2 = 1$. Set up the triple integrals in cylindrical coordinates that give the volume of D using the following orders of integration:
(i) $dz\, dr\, d\theta$, (ii) $dr\, dz\, d\theta$ and (iii) $d\theta\, dz\, dr$.

21. Let D be the region in the last exercise. Set up the triple integrals in spherical coordinates that give the volume of D using the following orders of integration.
(i) $d\rho\, d\phi\, d\theta$, (ii) $d\phi\, d\rho\, d\theta$.

22. Find the average value of the function $f(\rho, \phi, \theta) = \rho \cos\phi$ over the solid ball $\rho \leq 1$, $0 \leq \phi \leq \frac{\pi}{2}$.

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